Stat 512 — Take home exam II (due on July 15th)

1. Let $Y_1, \ldots, Y_n$ be independent Poisson random variables with means $\lambda_1, \ldots, \lambda_n$ respectively. Find:

   a. Probability function of $U = \sum_{i=1}^{n} Y_i$. (Hint: Using mgf technique.) (10 pts)

   b. Conditional probability function of Y_1, given that $U = m$. (Take a short review of conditional probability in 511). (10 pts)

2. If $Y_1, \ldots, Y_n$ are independent, uniformly distributed random variables on the interval $[0, \theta]$.

   a. Find joint density for $(Y_{(1)}, Y_{(n)})$. (10 pts)

   b. Find joint density for (U_1, U_2) where $U_1 = \dfrac{Y_{(1)}}{Y_{(n)}}$ and $U_2 = Y_{(n)}$. (15 pts)

   c. Are U_1 and U_2 independent? Briefly discuss your reason. (10 pts)

   d. If $\theta = 1$, show that $Y_{(k)}$, the kth-order statistic, has a beta distribution. Identify α and β. (15 pts)

3. Suppose that $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ are independent random samples, with the variables X_i normally distributed with mean μ_1 and variances σ_1^2 and the variables Y_i normally distributed with mean μ_2 and variances σ_2^2. The difference between the sample means, $\overline{X} - \overline{Y}$, is then a linear combination of $m + n$ normally distributed random variables and, is itself normally distributed.

   a. Find $E(\overline{X} - \overline{Y})$. (10 pts)

   b. Find var$(\overline{X} - \overline{Y})$. (10 pts)

   c. Suppose that $\sigma_1^2 = 2$ and $\sigma_2^2 = 2.5$, and $m = n$. Find the sample sizes m and n (hint: they are equal) so that $(\overline{X} - \overline{Y})$ will be within 1 unit of $(\mu_1 - \mu_2)$ with probability 0.95. (10 pts)

Extra credit. (10 pts)

4. Let Y_1 and Y_2 be independent and uniformly distributed over the interval $(0, 1)$. Find $P(2Y_{(1)} < Y_{(2)})$.